arXiv:hep-ph/9908455v1 24 Aug 1999

Reducing model dependence of spectator effects in inclusive decays of heavy baryons

M.B. Voloshin

Theoretical Physics Institute, University of Minnesota, Minneapolis, MN 55455

and

Institute of Theoretical and Experimental Physics, Moscow, 117259

Abstract

The dependence of inclusive weak decay rates of heavy hadrons on light spectator quarks is considered. The analysis of a previous work on relating the effects in b baryons to those in charmed baryons is extended to explicit evaluation of the matrix elements of certain four-quark operators over heavy baryons. It is shown that the usually postulated color antisymmetry of these matrix elements is significantly broken. The flavor-singlet shift of inclusive decay rates of b baryons due to the spectator effects is shown to be strongly suppressed in the leading-log approximation. Combined with the results for the flavor non-singlet splittings, this observation allows to argue, in a less model-dependent way, than before, that within the present understanding of the spectator effects it is highly unlikely that the lifetimes of the Λ_b and the B_d can be split by more than 10%.

1   Introduction

The differences in inclusive weak decay rates of hadrons, containing one heavy quark, are known [1] to be very prominent for the charmed mesons and baryons and, although are substantially smaller for the b hadrons, are well measurable and present a subject for an interesting experimental and theoretical study. Theoretically these differences are understood in the framework of the operator product expansion [2-5] in inverse powers of the heavy quark mass m_Q (for a later review see e.g. Ref. [6]). The leading term in this expansion is the 'parton' decay rate of the heavy quark $\Gamma_{part} \propto m_Q^5$ and does not depend on the light 'environment' surrounding the heavy quark Q in a hadron. Thus this term, setting the overall scale for the decay rates of hadrons containing the heavy quark Q, provides no splitting between the decay widths of specific hadrons. The first subleading term, suppressed with respect to the leading one by m_Q^{-2}, describes the effects of motion of the heavy quark in a hadron (time dilation) and the effects of the chromomagnetic interaction of the heavy quark [7]. This term distinguishes between mesons and baryons and between baryons of different spin structure, but provides no splitting of the decay rates within flavor SU(3) multiplets of heavy hadrons. The dependence on the spectator (anti)quark flavor arises in the next order, m_Q^{-3} relative to Γ_{part}, and is expressed through matrix elements of local four-quark operators over particular heavy hadron X_Q: $\langle X_Q | (\overline{Q}\,\Gamma\,Q)\,(\overline{q}\,\Gamma'\,q) | X_Q \rangle$ with different light quark flavors q and with different spin-color matrix structures Γ and Γ' [1]. It should be noted that although formally of higher order in m_Q^{-1}, the effects of the four-quark operators are generally larger than those of the $O(m_Q^{-2})$ terms for charmed hadrons and are larger than or comparable to the latter in the b hadrons, due to a large numerical factor.

The principal obstacle for quantitatively describing the spectator-dependent differences in the decay rates of heavy hadrons within this OPE-based theoretical scheme is our ignorance about the hadronic matrix elements of the relevant four-quark operators. For mesons one usually appeals to factorization and approximately expresses these matrix elements in terms of the mesons' annihilation constants f [2-6], with the thus arising uncertainty being relegated to the "bag constant" B. For the baryons the situation is even worse: most of the estimates operate with a naive (constituent, valence) quark model, where these matrix elements are expressed in terms of "wave function at the origin" $|\psi(0)|^2$, and then it is either assumed

---

[1]The physical origin of these terms is traditionally explained as due to the Pauli interference (PI) and the weak annihilation (WA)[2-6]. However in practical use of the OPE this terminology is somewhat redundant. Moreover, the physical interpretation of a particular four-quark operator as PI or WA depends on the hadron discussed.

that this quantity is the same as in mesons, or arguments for its suppression or enhancement in baryons are presented (see e.g. [3, 8, 9, 10]). Although this simplistic approach proved to be successful in qualitatively predicting the hierarchy of the lifetimes of charmed hadrons [5], its accuracy is clearly insufficient for a better quantitative description of the differences in the inclusive decay rates.

One of the problems, currently attracting a considerable interest, is presented by the measured difference of the lifetimes of the B mesons and of the Λ_b baryon [1]: $\tau(\Lambda_b)/\tau(B_d) = 0.81 \pm 0.05$. The effect of the $O(m_b^{-2})$ terms in this splitting is believed to be less than 2% [11, 12]. Thus, if confirmed by further measurements, this large difference in the lifetimes will have to be explained either by the contribution of the four-quark operators, or by effects beyond the OPE-based approach. So far all calculations based on the quark model approach as well as a calculation using QCD sum rules [13] have failed to find an enhancement of the four-quark matrix elements sufficient to achieve a 20% enhancement of the decay rate of Λ_b, required by the current data. The largest predictions for this enhancement go to only about 10%.

In view of this discrepancy between the available data and the calculations, it is highly desirable to reduce the model dependence of the theoretical predictions for the relevant four-quark matrix elements over heavy hadrons. One approach in this direction is to use available experimental data to evaluate the required matrix elements and then to apply the results to predict other differences in inclusive decay rates [14, 15]. In particular, in this way one can relate the differences in decay rates of the b hadrons to those for the charmed hadrons. Certainly, such approach is also not fully proof. One possible source of uncertainty is the application of the heavy quark expansion to charmed hadrons. The mass of charmed quark is by no means asymptotically heavy, thus subsequent terms in m_c^{-1} potentially can be large. One of obvious manifestations of a relatively low m_c is that the spectator effects in charmed hadrons are in fact dominating in the decay rates, rather than being small corrections. This however does not necessarily invalidate the applicability of the discussed OPE expansion to charmed hadrons, since the enhancement of the m_Q^{-3} terms by a large numerical factor has no recurrence in subsequent terms of the expansion. Another uncertainty arises from using the flavor SU(3) symmetry in order to relate matrix elements within one SU(3) multiplet. Both approximations, however can be tested by comparing the predictions within this approach [15] for inclusive decay rates into various channels (semileptonic and CKM suppressed), once the corresponding experimental data become available.

The purpose of the present paper is to extend the analysis of Ref. [15] of the decay

rate splittings in the triplet of heavy baryons (Λ_Q and two Ξ_Q's) to discussion of particular four-quark matrix elements over heavy baryons, and to an analysis of the overall (flavor SU(3)-singlet) shift of the inclusive decay rates for the baryonic triplet in addition to the splittings within the triplet.

As will be demonstrated here, although the data on the lifetimes of charmed baryons imply a significant enhancement (by a factor of 4-6) of the four-quark matrix elements, the color structure comes out entirely different from the expectations based on naive quark model. Namely, inherent in an naive quark model approach is the color antisymmetry relation for the baryonic matrix elements:

$$\langle X_Q | (\overline{Q}_i \, \Gamma \, Q^i) \, (\overline{q}_j \, \Gamma' \, q^j) | X_Q \rangle = -\langle X_Q | (\overline{Q}_i \, \Gamma \, Q^j) \, (\overline{q}_j \, \Gamma' \, q^i) | X_Q \rangle, \tag{1}$$

where this time Γ and Γ' stand for colorless spinor matrices. The color antisymmetry also holds in the sum rules calculation [13] within the level of complexity adopted there. Also this relation was used as an input in the most recent paper [16] on the subject. The relation (1) however can not be valid at all normalization scales μ for the operators due to different μ dependence of the color structures involved. Thus the validity and the meaning of the color antisymmetry are questionable at the least. It will be shown here that the data imply that the matrix element in the left-hand-side of eq.(1) is significantly enhanced, while that in the right-hand-side is rather suppressed and is likely to be negligible or zero at a low normalization point μ such that $\alpha_s(\mu) \sim 1$.

The enhancement of certain matrix elements, derived from the data on decays of charmed baryons, might inspire enthusiasm in explaining the data on $\tau(\Lambda_b)/\tau(B_d)$. Indeed, the result of Ref. [15] for the splitting of the total decay rates between Λ_b and Ξ_b^-: $\Gamma(\Lambda_b) - \Gamma(\Xi_b^-) = 0.11 \pm 0.03 \, ps^{-1}$, which constitutes $(14 \pm 4)\%$ of the measured width of Λ_b, might hint that shifts of the rates by the spectator effects by about 10 or 20% should not be uncommon among the b baryons[2]. However, as will be discussed, the splitting $\Gamma(\Lambda_b) - \Gamma(\Xi_b^-)$ turns out to be the largest spectator effect for the triplet of b baryons. The inclusive decay rates of Λ_b and Ξ_b^0 are degenerate up to terms additionally suppressed by m_c^2/m_b^2, while the overall (SU(3)-singlet) shift of the decay rates is greatly suppressed by a small coefficient in the flavor-singlet part of the effective Lagrangian describing the spectator effects. Parametrically, in the leading-log order (LLO), the suppression factor is proportional to $[(\alpha_s/\pi) \ln m_W/m_b]^2$, and (with

---

[2]Another very simplistic reasoning can be as follows. The spectator effects in the lifetimes of charmed baryons amount to few ps^{-1}. When rescaled to the b baryons by the factor $|V_{cb}|^2 \, m_c^2/m_b^2 \approx 0.015$, this puts the "natural" magnitude of these effects in the ballpark of $0.1 \, ps^{-1}$.

appropriate numerical factors) amounts to about 0.1. Thus the average shift of the decay rate in the baryon triplet is expected to be only of the order of $10^{-2}\,ps^{-1}$. When combined with the estimate of the chromomagnetic $O(m_b^{-2})$ effects and the prediction for the splitting of the rates within the b baryon triplet, the central value of the expected enhancement of the total decay width of Λ_b as compared to B_d does not exceed 8%. In view of the suppression of the discussed SU(3) singlet shift in the LLO approximation, the next-to-leading-log (NLLO) terms can in fact be as much essential as the LLO ones. However, it is highly unlikely that they can exceed the latter by a factor of more than 10. Thus we conclude that within an analysis, not dealing with a model of quark wave functions of heavy baryons, the central value of the current data on $\tau(\Lambda_b)/\tau(B_d)$ can not be accommodated in the framework of the OPE-based description of inclusive decay rates of heavy hadrons.

The rest of the paper is organized as follows. In Section 2 the OPE approach to spectator effects in inclusive decays of heavy hadrons is briefly described and the relevant to the reasoning in this paper parts of the effective Lagrangian with four-quark operators are presented. In Section 3 these expressions are used, continuing along the lines of Ref. [15], to extract certain combinations of the four-quark matrix elements over heavy baryons. In Section 4 the suppression in the leading-log order of the flavor-singlet shift of the decay rates of the (anti)triplet of the b baryons is demonstrated, and possible effects beyond that order are discussed in Section 5.

2 Spectator dependent terms in the OPE approach

The description of the leading in the limit $m_Q \to \infty$ as well as subleading effects in inclusive decay rates of heavy hadrons arises through application of operator product expansion to the 'effective Lagrangian' related to the correlator of the weak-interaction Lagrangian L_W:

$$L_{eff} = 2\,\mathrm{Im}\left[i \int d^4x\, e^{iqx}\, T\left\{L_W(x), L_W(0)\right\}\right] \;. \tag{2}$$

The inclusive decay rate of a heavy hadron X_Q is then determined as

$$\Gamma_X = \langle X_Q|\, L_{eff}\, |X_Q\rangle \;, \tag{3}$$

within the adopted throughout this paper non-relativistic normalization for the heavy quark states: $\langle Q|Q^\dagger Q|Q\rangle = 1$. The subject of interest in this paper, the dependence on the spectator quarks, is described by the term in the OPE for L_{eff}, containing the light quark fields. This term is denoted here as $L_{eff}^{(3)}$, indicating that it is the third term in the operator

4

expansion (after the leading one and those of order m_Q^{-2}). The expressions for the parts of $L_{eff}^{(3)}$ describing inclusive decay rates of the charmed and b hadrons into specific flavor-types of final states are found by picking the corresponding parts of the weak Lagrangian L_W. For the dominant unsuppressed by the CKM mixing nonleptonic decays of charmed hadrons, associated with the underlying quark process $c \to s\,u\,\overline{d}$, the relevant part of $L_{eff}^{(3)}$ reads as [5]

$$
\begin{aligned}
L_{eff,\,nl}^{(3,0)} = \cos^4\theta_c \, \frac{G_F^2\, m_c^2}{4\pi} \Big\{ & C_1\,(\overline{c}\Gamma_\mu c)(\overline{d}\Gamma_\mu d) + C_2\,(\overline{c}\Gamma_\mu d)(\overline{d}\Gamma_\mu c) + \\
& C_3\,(\overline{c}\Gamma_\mu c + \tfrac{2}{3}\overline{c}\gamma_\mu\gamma_5 c)(\overline{s}\Gamma_\mu s) + C_4\,(\overline{c}_i\Gamma_\mu c_k + \tfrac{2}{3}\overline{c}_i\gamma_\mu\gamma_5 c_k)(\overline{s}_k\Gamma_\mu s_i) + \\
& C_5\,(\overline{c}\Gamma_\mu c + \tfrac{2}{3}\overline{c}\gamma_\mu\gamma_5 c)(\overline{u}\Gamma_\mu u) + C_6\,(\overline{c}_i\Gamma_\mu c_k + \tfrac{2}{3}\overline{c}_i\gamma_\mu\gamma_5 c_k)(\overline{u}_k\Gamma_\mu u_i) + \\
& \tfrac{1}{3}\,\kappa^{1/2}\,(\kappa^{-2/9}-1)\left[2\,(C_+^2 - C_-^2)\,(\overline{c}\Gamma_\mu t^a c)\,j_\mu^a - (5C_+^2 + C_-^2)(\overline{c}\Gamma_\mu t^a c + \tfrac{2}{3}\overline{c}\gamma_\mu\gamma_5 t^a c)j_\mu^a\right]\Big\}
\end{aligned}
\tag{4}
$$

where i, k are color indices, $\Gamma_\mu = \gamma_\mu\,(1-\gamma_5)$, and the coefficients C_A, $A = 1,\ldots,6$ are given by

$$
\begin{aligned}
C_1 &= C_+^2 + C_-^2 + \tfrac{1}{3}(1-\kappa^{1/2})(C_+^2 - C_-^2)\,, \\
C_2 &= \kappa^{1/2}\,(C_+^2 - C_-^2)\,, \\
C_3 &= -\frac{1}{4}\left[(C_+ - C_-)^2 + \tfrac{1}{3}(1-\kappa^{1/2})(5C_+^2 + C_-^2 + 6C_+ C_-)\right]\,, \\
C_4 &= -\frac{1}{4}\,\kappa^{1/2}\,(5C_+^2 + C_-^2 + 6C_+ C_-)\,, \\
C_5 &= -\frac{1}{4}\left[(C_+ + C_-)^2 + \tfrac{1}{3}(1-\kappa^{1/2})(5C_+^2 + C_-^2 - 6C_+ C_-)\right]\,, \\
C_6 &= -\frac{1}{4}\,\kappa^{1/2}\,(5C_+^2 + C_-^2 - 6C_+ C_-)\,.
\end{aligned}
\tag{5}
$$

in terms of the well know short-distance renormalization factors C_+ and C_- for the non-leptonic weak interaction: $C_- = C_+^{-2} = (\alpha_s(m_c)/\alpha_s(m_W))^{4/b}$ with b being the coefficient in the one-loop beta function in QCD. For the case of the charmed quark decay one can use $b = 25/3$ (see e.g. in the textbook [17]). Furthermore, the parameter $\kappa = (\alpha_s(\mu)/\alpha_s(m_c))$ describes the so-called 'hybrid' renormalization of the operators below the heavy quark mass and down to a low normalization point μ, and $j_\mu^a = \overline{u}\gamma_\mu t^a u + \overline{d}\gamma_\mu t^a d + \overline{s}\gamma_\mu t^a s$ is the standard notation for the color current of light quarks with $t^a = \lambda^a/2$ being the generators of the color SU(3) group.

Besides the dominant nonleptonic decays, it is expected [14, 15] that the spectator effects in the once CKM suppressed nonleptonic and the semileptonic inclusive decay rates of charmed hyperons are quite large, and produce a non-negligible impact on the lifetimes of

5

these hyperons. Therefore in order to analyze the four-quark matrix elements from the data on the lifetimes of the charmed baryons, one has to consider also the corresponding terms in $L_{eff}^{(3)}$. The once CKM suppressed part, associated with the quark processes $c \to s\,u\,\overline{s}$ and $c \to d\,u\,\overline{d}$ has the form [15]

$$
L_{eff,\,nl}^{(3,1)} = \cos^2\theta_c\,\sin^2\theta_c\,\frac{G_F^2\,m_c^2}{4\pi}\,\{C_1\,(\overline{c}\Gamma_\mu c)(\overline{q}\Gamma_\mu q) + C_2\,(\overline{c}_i\Gamma_\mu c_k)(\overline{q}_k\Gamma_\mu q_i)+ \tag{6}
$$
$$
C_3\,(\overline{c}\Gamma_\mu c + \frac{2}{3}\overline{c}\gamma_\mu\gamma_5 c)(\overline{q}\Gamma_\mu q) + C_4\,(\overline{c}_i\Gamma_\mu c_k + \frac{2}{3}\overline{c}_i\gamma_\mu\gamma_5 c_k)(\overline{q}_k\Gamma_\mu q_i) +
$$
$$
2\,C_5\,(\overline{c}\Gamma_\mu c + \frac{2}{3}\overline{c}\gamma_\mu\gamma_5 c)(\overline{u}\Gamma_\mu u) + 2\,C_6\,(\overline{c}_i\Gamma_\mu c_k + \frac{2}{3}\overline{c}_i\gamma_\mu\gamma_5 c_k)(\overline{u}_k\Gamma_\mu u_i) +
$$
$$
\frac{2}{3}\,\kappa^{1/2}\,(\kappa^{-2/9} - 1)\,\left[2\,(C_+^2 - C_-^2)\,(\overline{c}\Gamma_\mu t^a c)\,j_\mu^a - (5C_+^2 + C_-^2)(\overline{c}\Gamma_\mu t^a c + \frac{2}{3}\overline{c}\gamma_\mu\gamma_5 t^a c)j_\mu^a]\}
$$

with the notation $(\overline{q}\,\Gamma\,q) = (\overline{d}\,\Gamma\,d) + (\overline{s}\,\Gamma\,s)$. The semileptonic part (per one lepton flavor), generated by the quark decays $c \to s\,\ell^+\,\nu$ and $c \to d\,\ell^+\,\nu$ is given by [14, 18, 19, 15]

$$
L_{eff,\,sl}^{(3)} =
$$
$$
\frac{G_F^2\,m_c^2}{12\pi}\,\{\cos^2\theta_c\,\left[L_1\,(\overline{c}\Gamma_\mu c + \frac{2}{3}\overline{c}\gamma_\mu\gamma_5 c)(\overline{s}\Gamma_\mu s) + L_2\,(\overline{c}_i\Gamma_\mu c_k + \frac{2}{3}\overline{c}_i\gamma_\mu\gamma_5 c_k)(\overline{s}_k\Gamma_\mu s_i)\right] +
$$
$$
\sin^2\theta_c\,\left[L_1\,(\overline{c}\Gamma_\mu c + \frac{2}{3}\overline{c}\gamma_\mu\gamma_5 c)(\overline{d}\Gamma_\mu d) + L_2\,(\overline{c}_i\Gamma_\mu c_k + \frac{2}{3}\overline{c}_i\gamma_\mu\gamma_5 c_k)(\overline{d}_k\Gamma_\mu d_i)\right] -
$$
$$
2\,\kappa^{1/2}\,(\kappa^{-2/9} - 1)\,(\overline{c}\Gamma_\mu t^a c + \frac{2}{3}\overline{c}\gamma_\mu\gamma_5 t^a c)j_\mu^a\}\ , \tag{7}
$$

with the coefficients L_1 and L_2 found as

$$
L_1 = (\kappa^{1/2} - 1), \qquad L_2 = -3\,\kappa^{1/2}\ . \tag{8}
$$

In order to relate the spectator effects in the b hadrons to those in the charmed ones, one also needs the expression for the part of $L_{eff}^{(3)}$ describing the b decays. It is sufficient at the present level of accuracy to retain only the term for the dominant nonleptonic decays, generated by the quark processes $b \to c\,\overline{u}\,q$ and $b \to c\,\overline{c}\,q$, where q stands for d or s. This part of $L_{eff}^{(3)}$ is given by [5]

$$
L_{eff,\,nl}^{(3,b)} = |V_{bc}|^2\,\frac{G_F^2\,m_b^2}{4\pi}\,\{\tilde{C}_1\,(\overline{b}\Gamma_\mu b)(\overline{u}\Gamma_\mu u) + \tilde{C}_2\,(\overline{b}\Gamma_\mu u)(\overline{u}\Gamma_\mu b)+
$$
$$
\tilde{C}_5\,(\overline{b}\Gamma_\mu b + \frac{2}{3}\overline{b}\gamma_\mu\gamma_5 b)(\overline{q}\Gamma_\mu q) + \tilde{C}_6\,(\overline{b}_i\Gamma_\mu b_k + \frac{2}{3}\overline{b}_i\gamma_\mu\gamma_5 b_k)(\overline{q}_k\Gamma_\mu q_i) +
$$
$$
\frac{1}{3}\,\tilde{\kappa}^{1/2}\,(\tilde{\kappa}^{-2/9} - 1)\,\left[2\,(\tilde{C}_+^2 - \tilde{C}_-^2)\,(\overline{b}\Gamma_\mu t^a b)\,j_\mu^a-\right.
$$
$$
\left.(5\tilde{C}_+^2 + \tilde{C}_-^2 - 6\,\tilde{C}_+\tilde{C}_-)(\overline{b}\Gamma_\mu t^a b + \frac{2}{3}\overline{b}\gamma_\mu\gamma_5 t^a b)j_\mu^a]\}\ , \tag{9}
$$

where again the notation $(\overline{q}\,\Gamma\,q) = (\overline{d}\,\Gamma\,d) + (\overline{s}\,\Gamma\,s)$ is used, and the 'tilde' over the renormalization coefficients denotes that these are determined as described above, albeit with $\alpha_s(m_b)$ instead of $\alpha_s(m_c)$. The symmetry between s and d quarks (U-spin symmetry) of the expression (9) is a result of the approximation, where a small kinematical difference of order m_c^2/m_b^2 between the two-body phase space of the quark pairs $c\overline{c}$ and $c\overline{u}$ is neglected. (The formula with the full kinematical factors can be found e.g. in [12].)

3 Flavor-nonsinglet matrix elements from differences of lifetimes of charmed baryons

When applied to inclusive decay rates of charmed baryons in the SU(3) (anti)triplet: $(\Lambda_c, \Xi_c^+, \Xi_c^0)$, the expressions in equations (4 - 7) allow one to extract matrix elements of certain flavor-nonsinglet four-quark operators from the data on the differences of lifetimes [15]. Indeed, using the property that there is no correlation of the spin of the heavy quark with its light 'environment' in these baryons, one finds that the operators with the axial current of the heavy quark do not contribute to the decay rates of the baryons, so that only the structures with the vector currents are relevant. These structures are of the type $(\overline{c}\,\gamma_\mu\,c)(\overline{q}\,\gamma_\mu\,q)$ and $(\overline{c}_i\,\gamma_\mu\,c_k)(\overline{q}_k\,\gamma_\mu\,q_i)$ with q being d, s or u. The difference among the baryons in the triplet of the inclusive decay rates: the dominant nonleptonic, the once CKM suppressed, and the semileptonic ones are then all determined in terms of two combinations of the matrix elements defined as [15]

$$x = \left\langle \frac{1}{2}\,(\overline{c}\,\gamma_\mu\,c)\left[(\overline{u}\,\gamma_\mu u) - (\overline{s}\,\gamma_\mu s)\right]\right\rangle_{\Xi_c^0 - \Lambda_c} = \left\langle \frac{1}{2}\,(\overline{c}\,\gamma_\mu\,c)\left[(\overline{s}\,\gamma_\mu s) - (\overline{d}\,\gamma_\mu d)\right]\right\rangle_{\Lambda_c - \Xi_c^+} , \quad (10)$$

$$y = \left\langle \frac{1}{2}\,(\overline{c}_i\,\gamma_\mu\,c_k)\left[(\overline{u}_k\,\gamma_\mu u_i) - (\overline{s}_k\,\gamma_\mu s_i)\right]\right\rangle_{\Xi_c^0 - \Lambda_c} = \left\langle \frac{1}{2}\,(\overline{c}_i\,\gamma_\mu\,c_k)\left[(\overline{s}_k\,\gamma_\mu s_i) - (\overline{d}_k\,\gamma_\mu d_i)\right]\right\rangle_{\Lambda_c - \Xi_c^+}$$

with the notation for the differences of the matrix elements: $\langle\mathcal{O}\rangle_{A-B} = \langle A|\mathcal{O}|A\rangle - \langle B|\mathcal{O}|B\rangle$.

The formulas for the differences within the charmed baryon triplet of inclusive rates for individual types of decay can be found in Ref. [15]. Here we make use of only the expressions for the differences of the total decay rates, $\Delta_1 \equiv \Gamma(\Xi_c^0) - \Gamma(\Lambda_c)$ and $\Delta_2 \equiv \Gamma(\Lambda_c) - \Gamma(\Xi_c^+)$, in terms of x and y:

$$\begin{aligned}
\Delta_1 &= \frac{G_F^2\,m_c^2}{4\pi}\cos^2\theta\left\{x\left[\cos^2\theta\,(C_5 - C_3) + \sin^2\theta\,(2\,C_5 - C_1 - C_3) - \frac{2}{3}\,L_1\right] + \right.\\
&\qquad\left. y\left[\cos^2\theta\,(C_6 - C_4) + \sin^2\theta\,(2\,C_6 - C_2 - C_4) - \frac{2}{3}\,L_2\right]\right\},
\end{aligned}$$

$$\Delta_2 = \frac{G_F^2\, m_c^2}{4\pi} \left\{ x \left[\cos^4\theta\,(C_3 - C_1) + \frac{2}{3}(\cos^2\theta - \sin^2\theta)\,L_1 \right] + \right.$$
$$\left. y \left[\cos^4\theta\,(C_4 - C_2) + \frac{2}{3}(\cos^2\theta - \sin^2\theta)\,L_2 \right] \right\}\,. \tag{11}$$

From these formulas one can express the matrix elements x and y in terms of the data on the lifetimes of the charmed hyperons. In doing so the values used here are $\Gamma(\Lambda_c) = 4.85 \pm 0.28\,ps^{-1}$, $\Gamma(\Xi_c^0) = 10.2 \pm 2\,ps^{-1}$, and the updated value [20] $\Gamma(\Xi_c^+) = 3.0 \pm 0.45\,ps^{-1}$. For evaluating the short-distance coefficients C_+ and C_- a realistic value $\alpha_s(m_c)/\alpha_s(m_W) = 2.5$ is used, and the results for the matrix elements only weakly depend on fine-tuning this ratio. As a result the μ independent matrix element x is found as

$$x = -(0.04 \pm 0.01)\,GeV^3 \left(\frac{1.4\,GeV}{m_c} \right)^2\,, \tag{12}$$

while the dependence of the thus extracted matrix element y on the normalization point μ is shown in Fig.1 [3].
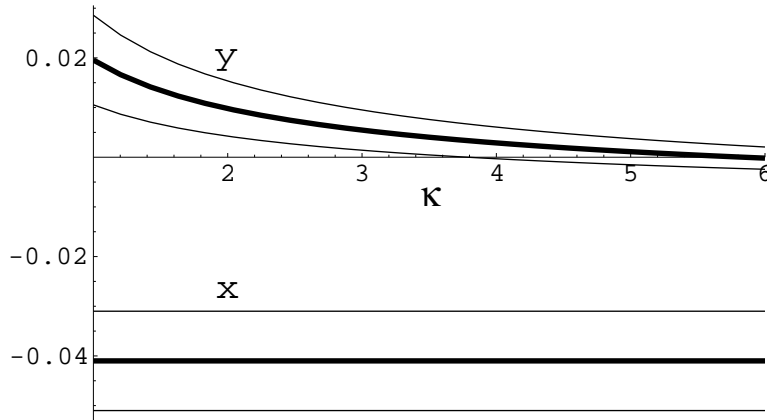

Figure 1: The values of the extracted matrix elements x and y in GeV^3 vs. the normalization point parameter $\kappa = \alpha_s(\mu)/\alpha_s(m_c)$. The thick lines correspond to the central value of the data on lifetimes of charmed baryons, and the thin lines show the error corridors. The extracted values of x and y scale as m_c^{-2} with the assumed mass of the charmed quark, and the plots are shown for $m_c = 1.4\,GeV$.

One can see that the extracted values of the matrix elements are in a drastic variance with naive models. Most conspicuously, the color antisymmetry relation $x = -y$ fails at

---

[3]It should be noted that the curves at large values of κ, $\kappa > \sim 3$, are shown only for illustrative purpose. The coefficients in the OPE, leading to the equations (11), are purely perturbative. Thus, formally, they correspond to $\alpha_s(\mu) \ll 1$, i.e. to $\kappa \ll 1/\alpha_s(m_c) \sim (3-4)$.

all μ less than m_c. Moreover, the trend in the plot of Fig.1 shows, that a possibly better approximation would be to assume $y \approx 0$ at a low scale μ close to the confinement scale. The implication of the vanishing of y is that at large distances there is no correlation between the color of an individual light quark and the heavy one, which looks quite natural, if one takes into account strong chaotization of the relative quark color due to emission of soft gluons. Furthermore, the absolute value of x significantly exceeds the simplistic estimate [4, 5], $f_D^2 m_D/12 \sim 0.006\, GeV^3$ of both $|x|$ and y in terms of the "wave function at the origin" in the baryons related to that in the mesons. In terms of the coefficient r introduced in [12] the result in eq.(12) for x corresponds to a quite large value $r \approx 5 \pm 1.5$.

4   Shifts of inclusive decay rates of b baryons

Once extracted from the data on the lifetimes of the charmed baryons, the matrix elements can be used for predicting the splitting of inclusive decay rates of the same baryons into subleading channels and also for predicting the splittings of the decay rates for the b baryons [15]. In particular, it might seem from the large value of x that the spectator effects in the b baryons can be relatively large on the appropriate for the b hadrons scale. Indeed, using the expressions for $L_{eff}^{(3)}$ one can directly relate the splitting of the decay rates $\Delta_b \equiv \Gamma(\Lambda_b) - \Gamma(\Xi_b^-)$ to the differences Δ_1 and Δ_2 for the charmed baryons [15]:

$$\Delta_b \approx |V_{bc}|^2 \frac{m_b^2}{m_c^2} \left(0.85\, \Delta_1 + 0.91\, \Delta_2\right) \approx 0.015\, \Delta_1 + 0.016\, \Delta_2 \approx 0.11 \pm 0.03\, ps^{-1}\ , \qquad (13)$$

which constitutes about 14% of the total decay rate of Λ_b. The decay rates of Λ_b and Ξ_b^0 are degenerate due to the approximation of the U symmetry, used in eq.(9). However, as will be argued in this section, the average shift of the decay rates in the b baryon triplet due to the considered spectator effects should be much smaller than this splitting and should amount to only of the order of $10^{-2}\, ps^{-1}$. Thus the enhancement of the decay rate of Λ_b by the four-quark effects amounts to essentially one third of the splitting in eq.(13).

In order to substantiate this claim we introduce the average decay rate in a heavy baryon triplet

$$\overline{\Gamma}_Q = \frac{1}{3} \left(\Gamma(\Lambda_Q) + \Gamma(\Xi_Q^1) + \Gamma(\Xi_Q^2)\right)\ , \qquad (14)$$

where Ξ_Q^1 and Ξ_Q^2 stand for the two heavy cascade hyperons (Ξ_c^0 and Ξ_c^+ if Q is the charmed quark, and Ξ_b^- and Ξ_b^0 in the case of b baryons). The contribution $\delta^{(3)}\overline{\Gamma}_Q$ of the four-quark

operators to $\overline{\Gamma}_Q$ is generally expressed in terms of two flavor-singlet matrix elements:

$$x_s = \frac{1}{3} \langle H_Q | (\overline{Q} \, \gamma_\mu \, Q) \left((\overline{u} \, \gamma_\mu \, u) + (\overline{d} \, \gamma_\mu \, d) + (\overline{s} \, \gamma_\mu \, s) \right) | H_Q \rangle$$

$$y_s = \frac{1}{3} \langle H_Q | (\overline{Q}_i \, \gamma_\mu \, Q_k) \left((\overline{u}_k \, \gamma_\mu \, u_i) + (\overline{d}_k \, \gamma_\mu \, d_i) + (\overline{s}_k \, \gamma_\mu \, s_i) \right) | H_Q \rangle \,, \tag{15}$$

where H_Q stands for a heavy hyperon in the (anti)triplet. Consider the contribution of the term in eq.(4) to the dominant nonleptonic part of the average decay rate in the charmed baryon triplet. Using the gamma matrix Fierz identities and the color Fierz identity $t_{ij}^a \, t_{kl}^a = \delta_{il} \, \delta_{kj}/2 - \delta_{ij} \, \delta_{kl}/6$, one finds for this contribution a simple expression:

$$\delta_{nl}^{(3,0)} \overline{\Gamma}_c = \cos^4 \theta \, \frac{G_F^2 \, m_c^2}{8\pi} (C_+^2 + C_-^2) \, \kappa^{5/18} \, (x_s - 3 \, y_s) \,. \tag{16}$$

Similarly, the average shift of the decay rate in the triplet of the b baryons is found from eq.(9) as

$$\delta^{(3)} \overline{\Gamma}_b = |V_{bc}|^2 \, \frac{G_F^2 \, m_b^2}{8\pi} (\tilde{C}_+ - \tilde{C}_-)^2 \, \tilde{\kappa}^{5/18} \, (x_s - 3 \, y_s) \,. \tag{17}$$

Therefore from the later two expressions one finds that in the ratio of the shifts the combination $(x_s - 3 \, y_s)$ of the unknown hadronic matrix elements cancels out, and that the shifts are related as

$$\delta^{(3)} \overline{\Gamma}_b = \frac{|V_{bc}|^2}{\cos^4} \, \frac{m_b^2}{m_c^2} \, \frac{(\tilde{C}_+ - \tilde{C}_-)^2}{C_+^2 + C_-^2} \left[\frac{\alpha_s(m_c)}{\alpha_s(m_b)} \right]^{5/18} \delta_{nl}^{(3,0)} \overline{\Gamma}_c \approx 0.0025 \, \delta_{nl}^{(3,0)} \overline{\Gamma}_c \,. \tag{18}$$

(One can observe, with satisfaction, that the dependence on the unphysical parameter μ also cancels out, as it should.) This equation shows that relatively to the charmed baryons the average shift of the decay rates in the b baryon triplet is greatly suppressed by the ratio $(\tilde{C}_+ - \tilde{C}_-)^2/(C_+^2 + C_-^2)$, which parametrically is of the second order in α_s, and numerically is only about 0.12.

An estimate of $\delta^{(3)} \overline{\Gamma}_b$ from eq.(18) in absolute terms depends on evaluating the average shift $\delta_{nl}^{(3,0)} \overline{\Gamma}_c$ for charmed baryons. The latter shift can be conservatively bounded from above by the average total decay rate of those baryons: $\delta_{nl}^{(3,0)} \overline{\Gamma}_c < \overline{\Gamma}_c = 6.0 \pm 0.7 \, ps^{-1}$, which then yields, using eq.(18), an upper bound $\delta^{(3)} \overline{\Gamma}_b < 0.015 \pm 0.002 \, ps^{-1}$. More realistically, one should subtract from the total average width $\overline{\Gamma}_c$ the contribution of the 'parton' term, which can be estimated from the decay rate of D_0 with account of the $O(m_c^{-2})$ effects [7], as amounting to about $3 \, ps^{-1}$. (One should also take into account the semileptonic contribution to the total decay rates, which however is quite small at this level of accuracy). Thus a realistic evaluation of $\delta^{(3)} \overline{\Gamma}_b$ does not exceed $0.01 \, ps^{-1}$.

10

5 Discussion

Due to the observed here suppression of the flavor-singlet spectator shift of the decay rates of the b baryons (eq.(17)), smaller effects, which are neglected so far, may become important in this quantity. One such effect is that of modification of the coefficients in the expression (9) for the spectator effects in nonleptonic decays of b baryons due to nonvanishing ratio m_c^2/m_b^2. By a simple inspection of the formula with full kinematical factors in Ref. [12] one can readily verify that the corresponding modification of the flavor-singlet part is of order $m_c^2/m_b^2 \sim 0.1$ with no anomalously large coefficients. Thus, the kinematical factors cannot significantly enhance the average shift of the decay rates in the baryon triplet. Another, more subtle, effect is that of the flavor SU(3) symmetry breaking. If it amounts to about 30% of the 'natural' value of the spectator effects in the b hadrons, it would obviously exceed the shift of the decay rates, given by eq.(18), however it would still be insufficiently large to explain the current data on $\tau(\Lambda_b)$. In view of the usual difficulty of theoretically estimating this effect, one can only rely on future data on the difference of lifetimes of Λ_b and Ξ_b^0, which would serve as a measure of both the $O(m_c^2/m_b^2)$ corrections in the coefficients and of the flavor SU(3) breaking in the spectator effects. One more source of corrections to the LLO formula in eq.(18) is provided by the next-to-leading-log terms in the perturbation theory coefficients in the OPE. Apriori these terms are parametrically suppressed by at least $\alpha_s(m_c)/\pi \sim 0.1$, i.e. the 'natural' magnitude of their contribution is of the same order as the LLO result in eq.(18). Whether these terms contain a large numerical enhancement can be found out only by explicit calculation.

Thus, summarizing the reasoning of the present paper, an analysis of spectator effects in heavy baryons, using the relations between the b baryon decays and the charmed baryon decays, rather than relying on models of baryon wave function, confirms the long-standing conclusion that theoretically it is highly unlikely that the ratio of the lifetimes $\tau(\Lambda_b)/\tau(B_d)$ is significantly below 0.9. Moreover, it is found that the shift due to spectator effects of the average decay rate of the triplet of the b baryons is considerably less than the splitting of the rates within the triplet, i.e. between the decay rate of Ξ_b^- and the (approximately) degenerate decay rates of Λ_b and Ξ_b^0. In other words, the predicted pattern of the lifetimes of these b hadrons is

$$\tau(\Xi_b^0) \approx \tau(\Lambda_b) < \tau(B_d) < \tau(\Xi_b^-) \tag{19}$$

with the difference between the largest and the smallest rates given by eq.(13). The degeneracy of the decay rates of Λ_b and Ξ_b^0 is a consequence of the approximations used, in particular

of the flavor SU(3) symmetry and of neglecting the kinematical corrections of order m_c^2/m_b^2. An experimental measurement of the difference of these rates would provide a fair measure of the validity of these approximations.

Acknowledgement

References

[1] Particle Data Group, Eur. Phys. J. **C 3** (1998) 1.

[2] M.A. Shifman and M.B. Voloshin, (1981) unpublished, presented in the review V.A.Khoze and M.A. Shifman, Sov. Phys. Usp. **26** (1983) 387.

[3] N. Bilic, B. Guberina and J. Trampetic, Nucl. Phys. **B248** (1984) 261.

[4] M.A. Shifman and M.B. Voloshin, Sov. J. Nucl. Phys. **41** (1985) 120.

[5] M.A. Shifman and M.B. Voloshin, Sov. Phys. JETP **64** (1986) 698.

[6] B. Blok and M. Shifman, *Lifetimes of Charmed Hadrons Revisited - Facts and Fancy.* In Proc. Workshop on the Tau-Charm Factory, p. 247, Marbella, Spain, 1993. Eds. J. Kirkby and R. Kirkby, Editions Frontiers, Gif-sur-Yvette, 1994.

[7] I.I. Bigi, N.G. Uraltsev, and A.I. Vainshtein, Phys. Lett. **B293** (1992) 430, erratum – ibid. **B297** (1993) 477.

[8] B. Guberina, R. Rückl, and J. Trampetic, Zeit. Phys. **C 33** (1986) 297.

[9] J.L. Rosner, Phys. Lett. **B 379** (1996) 267.

[10] N.G. Uraltsev, Phys. Lett. **B 376** (1996) 303.

[11] I.I. Bigi et. al., in *B Decays.* Ed. S. Stone, World Scientific, Revised Second Edition, 1994, p. 132.

[12] M. Neubert and C.T. Sachrajda, Nucl. Phys. **B483** (1997) 339.

[13] P. Colangelo and F. de Fazio, Phys. Lett. **B 387** (1996) 37.

[14] M.B. Voloshin, Phys. Lett. **B 385** (1996) 369.

[15] M. B. Voloshin, Preprint TPI-MINN-99/07-T, UMN-TH-1742-99. To be published in Phys. Rep. 1999; [hep-ph/9901445].

[16] B. Guberina, B. Melić and H. Štefančić, Preprint IRB-TH-4/99, July 1999; [hep-ph/9907468].

[17] L.B. Okun, *Leptons and Quarks*, North-Holland, Amsterdam, 1982, 1984.

[18] H.-Y. Cheng, Phys. Rev. **D56** (1997) 2783.

[19] B. Guberina and B. Melić, Eur.Phys.J. **C2** (1998) 697.

[20] Particle Data Group. *1999 WWW Update*, http://pdg.lbl.gov/1999/bxxx.html